Theory of fluctuations in high electric fields

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In the frame of real-time Green's-function formalism a generalized Lennard-Balescu equation is derived, which includes external field effects to arbitrary field strength as well as many-particle effects. This kinetic equation is gauge invariant and shows retardation effects connected with fields as well as collisional broadening. Consistent with this treatment the dielectric properties in nonequilibrium are described by the generalized dielectric function. From the kinetic equation the fluctuation-dissipation theorem is derived, which is strongly modified in the presence of high field strengths.

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I. INTRODUCTION

The problem of high-field transport in semiconductors has played an important role the past 30 years. Semiclassical studies are presented, e.g., in [1]. With increasing improvement of technological facilities the quantum transport effects have become more interesting [2]. The size of modern electronic devices is becoming continually smaller while applied voltages remain the same. This translates into very strong fields with rapid spatial variations. While the problem of high-field quantum transport [3] in strongly inhomogeneous conditions is far from being solved at present, the problem of high-field transport in uniform electric fields is more easily treated and some progress has been made recently [4,5]. A further motivation for the investigations of high electric fields is the influence of strong laser fields to matter [6,7]. The interactions of intense femtosecond laser pulses [8] give rise to surprising new physics at this time scale [9,6,10,11].

In this paper an attempt is made to bring together the high-field description of long-range Coulomb systems and quantum kinetic theory. The aim of the study presented here is the following. It is shown in a systematic way how dielectric properties and collision integrals can be derived on a consistent level of the random-phase approximation (RPA). This will lead to a nonequilibrium description of dielectric properties of long-range interactions of fermions or bosons resulting in a quantum Lennard-Balescu equation. Further, all these approximations are derived in the case of arbitrary high field strength and are considered therefore as a generalization of the theory, which was developed in [12]. First, a short overview of the development of kinetic equations without completeness is given.

The kinetic description, which started with the foundation of Boltzmann's famous equation 120 years ago [13], has been rapidly developed further from important classical contributions made by Chapman and Cowling [14], Enskog [15], Kirkwood [16], Bogoliubov [17], and Prigogine [18] to quantum extensions, where the pioneering work along these lines was done by Bogoliubov and Gurov [19] and Mori and Ono [20].

The general starting point in deriving kinetic equations is the coupled set of equations of motion for the reduced density operators. This set was first derived by Irving and Zwanzig [21]. The formal structure is similar to the so-called Bogoliubov-Born-Green-Kirkwood-Yvon (BBGKY) hierarchy [22] for reduced distribution functions in classical statistical physics. The first to use this BBGKY hierarchy in deriving kinetic equations were Bogoliubov [17], Born and Green [23], and Kirkwood [16]. The quantum Boltzmann equation differs from the classical one in the collision term, which takes into account that the final scattering states can be occupied and consequently blocked by the Pauli exclusion principle. Moreover, the quantum-mechanical transition rate is used rather than the classical one. Of the various other extensions proposed for the ordinary Boltzmann equation, the works of Klimontovich and Kremp [24] and McLennan [25] should be mentioned, which treat quantum gases with bound states. For a comprehensive overview see [26].

The most powerful and elegant techniques to describe the entire region of density and temperature as well as situations far from equilibrium, where linear response fails, is the method of the quantum-statistical Green's function. It was introduced in 1955 by Matsubara [27] for describing many-particle systems. The Green's functions used by him are also called imaginary-time Green's functions because their time arguments are only allowed to assume purely imaginary values. For this reason, they can only be used to describe equilibrium systems. In 1959, Martin and Schwinger [28] introduced Green's-function techniques for the description of many-body systems. As a starting point they constructed a hierarchy of equations of motion for the imaginary-time Green's functions, nowadays known as the Martin-Schwinger hierarchy.

A few years later, in 1962, Kadanoff and Baym [29] presented a complete transport theory based on these many-body techniques. An important component in the construction of this transport theory was an analytic continuation of the Green's function from imaginary times to real times. This step was necessary to describe nonequilibrium situations.

At about the same time Schwinger [30] initiated the use of real-time Green's functions. These functions are de-

fined on a directed contour and sometimes are also called path-ordered Green's functions. Both Keldysh [31] and Craig [32] used this concept and formulated, with the help of perturbative methods, a Dyson equation. The real-time Green's functions allow the properties of many-particle systems to be investigated in a ground state, in a finite-temperature equilibrium state, and in a nonequilibrium situation, in a consistent manner. Applications of real-time many-body methods range from problems of quantum chromodynamics [33], through nuclear physics [26,34–36], to theory of liquid helium [37], physics of plasmas [38], physics of condensed matter [39], astrophysics [40], and cosmology [41]. Thus it is indeed the most general formalism of many-particle theory we have today.

The outline of the paper is the following. A brief rederivation of the basic equations appropriate for the coupling of high electric fields is given in Sec. II. The main problem is to formulate the gauge-invariant equations [42–44]. This is done in Sec. IV, with the result of easily applicable rules.

Once the general equations are laid out, the self-energy has to be determined, which is the central quantity in the theory. Kadanoff and Baym [29] introduced the two fundamental approximations, the so-called screened potential and the T-matrix approximation, which are still the basic equations of nonequilibrium Green's-function technique. Any subsequent studies merely provided an alternative derivation either of the kinetic equation itself or of the T-matrix equation, the most elaborate of which was given by Danielewicz [45]. This derivation mostly based itself on a simple approximation by means of a perturbation series. However, in doing so, it was not clear what this approximation implied for higher-order Green's functions. This method was extended by Kremp, Schlanges, and Bornath [46,47] and later on was used by Danielewicz [48] as well as Botermans and Malfliet [26]. For the screened potential approximation a general treatment was given in [49], yielding a generalized Lennard-Balescu equation.

The aim of Sec. III is to show how the screened potential approximation can be generalized to include arbitrarily high external electric fields. Sec. V A then presents the resulting general kinetic equation. This kinetic equation is valid for interacting particles in arbitrarily high electric fields describing the long-range density fluctuations. The explicit non-Markovian behavior of the resulting collision integral is a generalization of the first Born approximation to an infinitesimal sum of interactions. This is an extension of the results in [12] and [49].

II. REAL-TIME GREEN'S-FUNCTION TECHNIQUE

We want to consider a system of charged particles of fermions or bosons under the influence of an applied constant electric field. Coupling of the field in vector potential gauge we have the Hamiltonian

$$H = \sum_i \int d\mathbf{r}\, \Psi_i^*(\mathbf{r},t)\, H_i\left(\frac{\hbar}{i}\nabla_{\mathbf{r}} - \frac{e}{c}\mathbf{A}(t)\right)\Psi_i(\mathbf{r},t) + \frac{1}{2}\sum_{i,j}\int d\bar{\mathbf{r}}d\mathbf{r}\, \Psi_i^*(\mathbf{r},t)\,\Psi_j^*(\bar{\mathbf{r}},t) \times V_{i,j}(\mathbf{r}-\bar{\mathbf{r}})\,\Psi_i(\mathbf{r},t)\,\Psi_j(\bar{\mathbf{r}},t)\ . \quad (1)$$

With Coulomb interaction V

$$V_{ij}(\mathbf{r}-\bar{\mathbf{r}}) = \frac{e^2 Z_i Z_j}{|\mathbf{r}-\bar{\mathbf{r}}|}.$$

A. Definition and equation of motion

In order to describe correlations in highly nonequilibrium situations, we define various correlation functions by different products of creation and annihilation operators

$$g^>(1,2) = \frac{1}{i}\langle\Psi(1)\Psi^*(2)\rangle$$
$$g^<(1,2) = \mp\frac{1}{i}\langle\Psi^*(2)\Psi(1)\rangle. \quad (2)$$

Here the $\langle\,\rangle$ denotes the average value with the unknown statistical *nonequilibrium* operator ρ and the numbers $(1,2\ldots)$ correspond to the variables $(\mathbf{r},\mathbf{R},\mathbf{s},\ldots,t)$.

From the above definitions we can build a causal function with the help of the step function Θ,

$$g(1,2) = \Theta(t_1-t_2)g^>(1,2) - \Theta(t_2-t_1)g^<(1,2). \quad (3)$$

Furthermore, it is useful to introduce the retarded and advanced values

$$g^{r/a}(1,2) = \pm\Theta[\pm(t_1-t_2)]\,[g^>(1,2)-g^<(1,2)]. \quad (4)$$

Instead of determining the nonequilibrium statistical operator we follow another concept of statistical mechanics and use the equation of motion of the creation and annihilation operators to derive kinetic equations, which may be solved with the appropriate choice of boundary and initial conditions. Applying the equation of motion for the field operators in the Heisenberg picture, one finds the famous Martin-Schwinger equation hierarchy [28], where the one-particle Green's function couples to the two-particle one, etc.

A formally closed equation can be reached with the introduction of the self-energy

$$\mp\int d2V(1-2)g_2(12,1'2^+) = \int d\bar{1}\Sigma(1,\bar{1})g_1(\bar{1},1')\ . \quad (5)$$

The self-energy can be split into two parts [46]:

$$\Sigma(1,1') = \Sigma_{\rm HF}(1,1') + \Sigma_C(1,1') \quad (6)$$

with the so called Hartree-Fock part

$$\Sigma_{\rm HF}(1,1') = \left(\mp i\delta(r_1-r_1')\int dr_2 V(r_1-r_2)g_1^<(r_2t_1'r_2t_1) + iV(r_1-r_1')g_1^<(r_1t_1r_1't_1')\right)\delta(t_1-t_1') \quad (7)$$

and the correlation part

$$\Sigma_C(1,2) = \Theta(t_1 - t_2)\Sigma_C^{>}(1,2) - \Theta(t_2 - t_1)\Sigma_C^{<}(1,2). \quad (8)$$

In order to obtain solutions and the way of integration it is necessary to specify the initial conditions. In many physical situations, the condition of weakening of initial correlation is an appropriate choice

$$\lim_{t_1 \to -\infty} g_2(121'2')|_{t_1 = t_2 + \epsilon} = g(11')g(22') - g(12')g(21') \; . \quad (9)$$

From this condition we get

$$\int_C d\bar{1}\Sigma_C(1,\bar{1})g(\bar{1},1') = \int_{-\infty}^{+\infty} d\bar{1}\left\{\Sigma_c(1,\bar{1})g(\bar{1},1') - \Sigma_c^{<}(1,\bar{1})g^{>}(\bar{1},1')\right\} . \quad (10)$$

It is easy to see that the boundary condition is fulfilled since the contribution (10) vanishes in the limit $t_1' = t_1^{\pm} \to -\infty$. For the case $t_1 < t_1'$ we can write, e.g.,

$$\int_{-\infty}^{+\infty} d\bar{1}\left\{\Sigma(1,\bar{1})g(\bar{1},1') - \Sigma^{<}(1,\bar{1})g^{>}(\bar{1},1')\right\} = \int_{-\infty}^{t_1} \Sigma^{>}(1,\bar{1})g^{<}(\bar{1},1') + \int_{t_1}^{t_1'} \Sigma^{<}(1,\bar{1})g^{<}(\bar{1},1') + \int_{t_1'}^{\infty} \Sigma^{<}(1,\bar{1})g^{>}(\bar{1},1') - \int_{-\infty}^{\infty} \Sigma^{<}(1,\bar{1})g^{>}(\bar{1},1') \; , \quad (11)$$

which vanishes for $t_1 \to t_1 \to t_0 = -\infty$. If we split the last integral on the right-hand side of (11) into two parts according to

$$\int_{-\infty}^{+\infty} d\bar{t}_1 = \int_{-\infty}^{t_1'} d\bar{t}_1 + \int_{t_1'}^{-\infty} d\bar{t}_1 \; ,$$

a contour of time integration follows that is equal to the Keldysh contour. In other words, this means that the weakening of initial correlation, which breaks the time-reversal symmetry, is equivalent to the Keldysh contour integration. With the expressions (5) and (10) we can finally write the first equation of the Martin-Schwinger hierarchy in the form

$$\left(i\frac{\partial}{\partial t_1} + \frac{[\frac{\hbar}{i}\nabla_1 - \frac{e}{c}A(t_1)]^2}{2m}\right) g_1(1,1') = \delta(1-1') - i\int_c d2V(1-2)g_2(121'2^+) = \int_C d\bar{1}\Sigma_c(1,\bar{1})g(\bar{1},1') = \int_{-\infty}^{+\infty} d\bar{1}\left\{\Sigma_c(1,\bar{1})g(\bar{1},1') - \Sigma_c^{<}(1,\bar{1})g^{>}(\bar{1},1')\right\} \; . \quad (12)$$

With the help of the definition of the causal Green's function (3), the Kadanoff-Baym equation [29,31] is obtained

$$\left(i\frac{\partial}{\partial t_1} + \frac{[\frac{\hbar}{i}\nabla_1 - \frac{e}{c}A(t_1)]^2}{2m}\right) g_1^{\gtrless}(1,1') = \int d\bar{r}_1 \Sigma_{\mathrm{HF}}(r_1,\bar{r}_1,t_1)g_1^{\gtrless}(\bar{r}_1,t_{1'}1') + \int_{-\infty}^{t_1} d\bar{1}\left[\Sigma_C^{>}(1,\bar{1}) - \Sigma_C^{<}(1,\bar{1})\right] g_1^{\gtrless}(\bar{1},1') - \int_{-\infty}^{t_1'} d\bar{1}\Sigma_C^{\gtrless}(1,\bar{1})\left[g_1^{>}(\bar{1},1') - g_1^{<}(\bar{1},1')\right] \; . \quad (13)$$

Subtracting this equation from its conjugate and introducing Wigner coordinates $T = (t_1 + t_2)/2, \tau = t_1 - t_2$, etc., we finally obtain the time diagonal part for space homogeneous systems [50]

$$i\frac{\partial}{\partial T}f_W(p,T) = \int_{-\infty}^{0} d\tau\left\{g^{>}\left(p,\tau,T-\frac{\tau}{2}\right),\Sigma^{<}\left(p,-\tau,T-\frac{\tau}{2}\right)\right\} - \left\{g^{<}\left(p,\tau,T-\frac{\tau}{2}\right),\Sigma^{>}\left(p,-\tau,T-\frac{\tau}{2}\right)\right\} . \quad (14)$$

Here $f_W(p,R,T) = \mp ig^{<}(p,R,T,\tau = 0)$ denotes the Wigner distribution function and $\{\,,\,\}$ is the anticommutator over integrals of Wigner coordinates indicated on the right-hand side. This equation is exact in time, but according to the assumed space homogeneity we used gradient expansion for space variables and dropped all R dependence for simplicity.

B. The Langreth-Wilkins rules

With the help of (9) it is possible to establish useful algebraic rules known as Langreth-Wilkins rules, which describe the way to obtain correlation or retarded functions from causal ones. They were first developed by Langreth and Wilkins [51]. For example, if we have products of Green's functions and integration over inner variables

$$C(1,2) = \int d\bar{1}A(1,\bar{1})B(\bar{1},2) \; , \quad (15)$$

we can show that, with the help of (9), the following rules can be obtained, building up correlation functions from (15):

$$C^{\gtrless}(1,2) = \int d\bar{1}A^r B^{\gtrless} + \int d\bar{1}A^{\gtrless}B^a \; , \quad (16)$$

$$C^{r/a}(1,2) = \int d\bar{1}A^{r/a}B^{r/a}. \quad (17)$$

Using these relations, one can derive the following generalized Kadanoff-Baym equations from (12) by using the inverse interaction free Green's function g_0^{-1} [50]:

$$\left[g_0^{-1} - \mathrm{Re}\Sigma, g^{\gtrless}\right] = \left[\Sigma^{\gtrless}, \mathrm{Re}g\right] + \frac{1}{2}\left\{g^{<},\Sigma^{>}\right\} - \frac{1}{2}\left\{g^{>},\Sigma^{<}\right\} . \quad (18)$$

Here [,] and {,} denote the commutator and anticommutator of integrals (15), respectively. This result is a compact written form of the Kadanoff-Baym equation. Nevertheless, it is sometimes advantageous to use this operator notation in order to investigate properties beyond the quasiparticle picture [26,52,53]. From this operator equation it is possible to derive the time diagonal equation (14) as well [50,5].

III. DENSITY FLUCTUATIONS

In order to describe fluctuations in the nonequilibrium many-particle system in terms of Green's functions, it is useful to introduce the correlation function [49]

$$L(121'2') = i[g_2(121'2') - g_1(11')g_1(22')]. \tag{19}$$

The density fluctuation is given by

$$\delta\rho(11') = \Psi^+(1')\Psi(1) - \langle\Psi^+(1')\Psi(1)\rangle. \tag{20}$$

Now we consider the special function

$$L(121^+2^+) = L(12)\ , \tag{21}$$

which may be written as

$$L(12) = \Theta(t_1 - t_2)L^>(12) + \Theta(t_2 - t_1)L^<(12). \tag{22}$$

It follows immediately that the correlation function of density fluctuations is given by

$$\langle\delta\rho(11^+)\delta\rho(22^+)\rangle = iL^>(12) = iL^<(21). \tag{23}$$

Apart from the density fluctuations, it is also useful to consider the field fluctuations, which may be expressed in momentum space as

$$\delta\mathbf{E}(k) = -i\frac{\epsilon_0\mathbf{k}}{e}\delta n(k).$$

For these fluctuations it is useful to define a symmetrized correlation function by

$$\overline{\langle\delta\mathbf{E}(1)\delta\mathbf{E}(2)\rangle} = \frac{1}{2}[\langle\delta\mathbf{E}(1)\delta\mathbf{E}(2)\rangle + \langle\delta\mathbf{E}(2)\delta\mathbf{E}(1)\rangle]$$
$$= \frac{\epsilon_0^2k^2}{e^2}\frac{L^>(12) + L^<(12)}{2}. \tag{24}$$

In order to derive an equation of motion, we apply the differential operator from the Martin-Schwinger hierarchy to the definition of L (19). With the help of the second equation of the Martin-Schwinger hierarchy, which couples the two-particle Green's function to the three-particle one, we obtain the following equation for L:

$$-i\left(i\frac{\partial}{\partial t_1} + \frac{[\frac{\hbar}{i}\nabla_1 - \frac{e}{c}A(t_1)]^2}{2m}\right)L(121'2') = \delta(1-1')G(22') - \delta(1-2')G(21') - \delta(1-1')G(22')$$
$$+G(22')\int d3V(13)L(131'3^+) + \int d3V(13)G(33^+)L(121'2')$$
$$+iG(11')G(22')\int d3V(13)G(33') - i\int d3V(13)G_3(1231'2'3^+). \tag{25}$$

Instead of taking into account binary collision, which describes short-ranged potentials, we are now interested in systems where long-ranged Coulomb potentials are important. Therefore, other approximations are necessary, which take into account collective effects (shielding). Such effects can be described in a convenient manner in terms of the defined fluctuation quantities. Therefore, let us consider the three-particle Green's function in the approximation [49]

$$iG_3(1231'2'3^+) = iG(11')G(22')G(33')$$
$$+G(11')L(232'3^+) + G(22')L(131'3^+)$$
$$+G(33^+)L(121'2'), \tag{26}$$

which describes the three-particle interaction for all possible two-particle fluctuations. This approximation will turn out to be the RPA approximation. For the equation (25) of L we now obtain a closed equation

$$\left(i\frac{\partial}{\partial t_1} + \frac{[\frac{\hbar}{i}\nabla_1 - \frac{e}{c}A(t_1)]^2}{2m})L(121'2'\right)$$
$$= -i\delta(1-2')G(21')$$
$$-i\int d3V(13)L(232'3^+)G(11'). \tag{27}$$

The occurring integrations should now be specified and the appropriate solutions selected. Therefore, the condition of the weakening of the initial correlation (9) is rewritten by the definition of L (19) into the form

$$L(121^+2^+)|_{t_1\to-\infty} = \mp iG(12^+)G(21^+) = L_0(121^+2^+). \tag{28}$$

The equation (27) of motion for L can then be solved by neglecting higher orders of interaction in the following manner:

$$L(421'2') = L_0(421'2') + L_0(411'1^+)V(13)L(232'3^+). \tag{29}$$

We note the special case of Eq. (29) for the two-point function (21)

$$L^{ab}(12) = L_0^{aa}(12)\delta^{ab} + \sum_c L_0^{aa}(14)V^{ac}(43)L^{cb}(32). \tag{30}$$

The different kinds of particles have been explicitly marked by latin letters. Obviously, the relation $L_0^{ab} \sim \delta^{ab}$ holds true. In the following, we will drop this notation for

simplicity and remember the right notation at the end.

Equation (30) is a causal one, which means that all functions entering are causal ones. Therefore, we can apply the Langreth-Wilkens rules to end up with

$$L^{\gtrless} = L_0^{\gtrless} + L_0^R V L^{\gtrless} + L_0^{\gtrless} V L^A \,, \qquad L^{R/(A)} = L_0^{R/(A)} + L_0^{R/(A)} V L^{R/(A)}. \tag{31}$$

Using the second equation one gets from the first equation the optical theorem

$$L^{\gtrless} = (\epsilon^R)^{-1} L_0^{\gtrless} (\epsilon^A)^{-1} \,, \tag{32}$$

where we introduced the operators of retarded and advanced dielectric functions, whose most general form in many-particle theory is given by

$$\epsilon^R = 1 - L_0^R V \equiv 1 + V L^R, \qquad \epsilon^A = 1 - V L_0^A \equiv 1 + L^A V. \tag{33}$$

The discussion of this nonequilibrium dielectric function and the resulting instability can be found in [54,55]. At this point we see that from the RPA equation (31) no initial correlation terms follow. This means that the condition of weakening of initial correlations (28) may be poor in the case of particle-plasmon interactions because there one needs an additional equation for the nonthermal plasmons [10,6]. Here we are restricted to particle-particle interactions and the corresponding density-density fluctuations in arbitrary high fields.

With the help of all these tools we are now able to write down the form of the self-energy. From the definition of L in (19) and the introduction of self-energy (5), one sees that the following relation holds true:

$$\mp \int d2V(12) L(121'2^+) = \int \Sigma'(12) G(21'). \tag{34}$$

Here Σ' denotes the self-energy minus the Hartree part. The Fock part can be split further to obtain the correlated self-energy

$$\Sigma'(12) = \Sigma_c(12) \pm iV(12) g^<(12).$$

The required correlation self-energy is then easily derived in the following way:

$$\Sigma_{ab}^{\gtrless}(11') = i \sum_c \int d2d2' V_{ab}(12) L_{bc}^{\gtrless}(2'1') V_{ca}(2'1') g_a^{\gtrless}(11'). \tag{35}$$

This expression will serve as a starting point for discussions in Sec. V, where we derive kinetic equations with field dependence.

IV. SPECTRAL PROPERTIES

In order to describe high-field effects and many-particle behavior correctly, it is necessary to determine the right kinetic equation. One fundamental problem is the inclusion of all field effects and approximations for many particle effects which do not touch the validity of the obtained equations to any field strength. It is known that the gradient expansion in time corresponds to a linearization of fields [56]. To get an unambiguous way of constructing approximations we have to formulate our gauge-invariant theory. This can be done by following a procedure known from field theory [42] and applied to high-field problems in [43].

One can introduce a gauge-invariant Fourier transform of the difference coordinate x,

$$\bar{g}(k,X) = \int \frac{dx}{(2\pi\hbar)^4} \times \exp\left(\frac{i}{\hbar} \int_{-\frac{1}{2}}^{\frac{1}{2}} d\lambda \, x_\mu [k^\mu + \frac{e}{\hbar c} A^\mu (X + \lambda x)] \right) \times g(xX). \tag{36}$$

For constant electric fields, which will be of interest in the following, one obtains, for the generalized Fourier-transform,

$$\bar{g}(k,X) = \int \frac{dx}{(2\pi\hbar)^4} \, e^{\frac{i}{\hbar}[x_\mu k^\mu - e\mathbf{r}\cdot\mathbf{E}T]} g(x,X),$$

where the χ function was chosen in such a way as that the scalar potential is zero $A^\mu = (0, -c\mathbf{E}T)$. Therefore, we have the following rules in formulating the gauge-invariant kinetic theory: (i) The difference variable x Fourier transfors to the canonical momentum p. (ii) The momentum shifts to the kinematical momentum according to $p = k - eET$. (iii) The gauge-invariant functions $\bar{g}$ are given by

$$g(p,T) = g(k - eET, T) = \bar{g}(k,T) = \bar{g}(p + eET, T). \tag{37}$$

We will make use of these rules in the following sections.

A. Free particles in a uniform electric field

We want to consider next the retarded Dyson equation for a particle in a uniform electric field, where the field is represented by a vector potential $E(t) = -c\,\dot{A}(T)$

$$\left[i\frac{\partial}{\partial t} - \epsilon\left(p - \frac{e}{c}A(t)\right) \right] g_0^R(p, tt') = \delta(t - t'). \tag{38}$$

This equation is readily integrated [5,57]

$$g_0^R(p, tt') = -i\Theta(t - t') \exp\left[-i \int_t^{t'} du \, \epsilon[p - A(u)] \right]. \tag{39}$$

For free particles corresponding to parabolic dispersions the gauge-invariant spectral function [5,57] follows:

$$a_0(k,\omega) = 2\int_0^\infty d\tau \cos\frac{1}{\hbar}\left(\hbar\omega\tau - \frac{k^2}{2m}\tau - \frac{e^2E^2}{24m}\tau^3 \right) = \frac{2\pi}{\epsilon_E} \mathrm{Ai}\left(\frac{k^2/2m - \hbar\omega}{\epsilon_E} \right) \,, \tag{40}$$

where $\mathrm{Ai}(x)$ is the Airy function [58] and we defined $\epsilon_E = (\hbar^2 e^2 E^2/8m)^{1/3}$. It is instructive to verify that (40) satisfies the frequency sum rule.

Let us return to the interaction-free retarded function (39). In frequency space, it reads

$$g_0^R(k,\omega) = -\int \frac{d\bar{\omega}}{2\pi\epsilon_E} \frac{\mathrm{Ai}\left(\frac{k^2/2m-\hbar\omega-\hbar\bar{\omega}}{\epsilon_E}\right)}{\hbar\omega - i\epsilon}. \quad (41)$$

Applying the Airy function orthogonality [59], the following relation holds true:

$$2\pi\int_{-\infty}^{\infty} \frac{d\omega}{\epsilon_E} g_0^R(k,\omega')\mathrm{Ai}\left(\frac{\hbar\omega-\hbar\omega}{\epsilon_E}\right) = \frac{1}{\omega\hbar - \frac{k^2}{2m} + i\epsilon} = g_0^R(E=0,k,\omega). \quad (42)$$

This means, in other words, that the interaction-free but field-dependent retarded Green's function g_0^R can be obtained from the interaction-free and field-free Green's function by a simple Airy transformation. This is an expression of the fact that the eigenfunctions of the Schrödinger equation with constant electric field are Airy functions and the retarded functions (4) can therefore be diagonalized with those eigensolutions [60,43].

B. The quasiparticle picture

Next we want to include collisions, which can be done by the Dyson equation with self-energy

$$\left(i\hbar\frac{\partial}{\partial t_1} + \frac{(\frac{\hbar}{i}\nabla_1 + eEt_1)^2}{2m}\right) g^R(1,1') = \delta(1-1') + \int d\bar{1}\Sigma^R(1,\bar{1})g^R(\bar{1},1') \ . \quad (43)$$

From this equation it is now possible to derive an *exact* solution for the *inverse* retarded Green's function. First we see from (43) the following form of the inverse functions, which may be understood as operators in space-time when the internal integration is performed:

$$(g^R)^{-1}(rR\tau T) = -\left[i\hbar\frac{1}{2}\frac{\partial}{\partial T} - i\hbar\frac{\partial}{\partial\tau} + \frac{[\frac{\hbar}{i}\frac{1}{2}\nabla_R - \frac{\hbar}{i}\nabla_r - eE(T-\frac{\tau}{2})]^2}{2m}\right]\delta(\tau)\delta(r). \quad (44)$$

Here we have introduced Wigner coordinates following $\tau = t - \bar{t}$ and $T = (t+\bar{t})/2$. If one adds the conjugate equation, one finds in the momentum-frequency domain

$$(g^R)^{-1}(pR\omega T) = \left[\omega\hbar - \frac{(p+eET)^2}{2m}\right] - \Sigma^R(pR\omega T), \quad (45)$$

which means that (45) is an exact form of the inverse operator of retarded functions including any interaction, whereby no approximations were made. The problem is merely to find the function itself, since the knowledge of the retarded one provides the spectral function and thus the spectral properties of the system. It can finally be stated that the form of (45) is easily written in gauge-invariant form (see Sec. IV), which is as well an *exact* relation

$$(g^R)^{-1}(kR\omega T) = \left[\omega\hbar - \frac{k^2}{2m}\right] - \Sigma^R(kR\omega T) \ ,$$

where all quantities are then gauge invariant. It is not possible, however, to derive the required inverse of this form simply by inverting the right-hand side and considering the frequency singularity placed on the upper half plane corresponding to the retarded character. As we demonstrated in Sec. IV A, this would merely yield the interacting yet field-free spectral function. Instead, we have to transform additionally with the Airy transformation to get the correct field-dependent spectral function (42).

With the help of this procedure we include now the many-particle effects in the quasiparticle picture. Therefore, we follow a standard procedure for the field-free case. With conventional gradient expansion, one can invert the *field-free* Dyson equation (43) directly and obtain in the *second-order* gradient expansion the spectral function

$$a_{E=0}(p\omega RT) = \frac{\mathrm{Im}\Sigma^R(p\omega RT)}{[\omega\hbar - \frac{p^2}{2m} - \mathrm{Re}\Sigma^R(p\omega RT)]^2 + [\frac{1}{2}\mathrm{Im}\Sigma(p\omega RT)]^2}. \quad (46)$$

For small imaginary parts of self-energy, and consequently small damping, the δ function introducing the quasiparticle picture is derived

$$a_{E=0}(p\omega RT) = 2\pi\delta\left(\frac{p^2}{2m} + \mathrm{Re}\Sigma^R(p\omega RT)\right) = \frac{2\pi}{|1 - \partial_\omega \mathrm{Re}\Sigma(p\omega RT)|_{\omega=\epsilon}}\delta(\omega\hbar - \epsilon(pRT)), \quad (47)$$

with the quasiparticle energy ϵ, which can be found from (47) by the dispersion relation

$$\omega\hbar - \frac{p^2}{2m} - \mathrm{Re}\Sigma^R(p\omega RT) = 0. \quad (48)$$

This is the required approximation for the many-particle influence.

As long as it is justified to introduce a quasiparticle picture by vanishing damping, we find the *field-free* spectral function as a sharp peak around the quasiparticle energies, which are now *independent of* ω. This means that we can apply the transformation (42) to obtain the field-dependent spectral function including many-particle influences. In a convenient time domain one finds

$$a_E(k\tau RT) = \exp\left[-\frac{i}{\hbar}\left(\epsilon_k\tau + \frac{e^2E^2}{24m}\tau^3\right)\right] ,$$

or in frequency space

$$a_E(k\omega RT) = \frac{2\pi}{\epsilon_E}\mathrm{Ai}\left(\frac{1}{\epsilon_E}(\omega\hbar - \epsilon_k)\right) \tag{49}$$

with $k = p - eET$. This is the main result of this section. The derived spectral function will be called a *joint spectral function*, according to [57]. It is the natural generalization of the quasiparticle picture to high-field situations and takes both effects into account: the gauge invariance and the many-particle influence. It might be informative to note that the dispersion relation (48) itself is gauge invariant.

C. The problem of the ansatz

In order to close the kinetic equation (14), it is necessary to know the relation between $g^>$ and $g^<$. This problem is known as an ansatz and must be constructed consistently with the required approximation of self-energy. The conventional way to do this is to change the correlation functions into the generalized distribution function and into the spectral one, which is an exact transformation

$$\begin{aligned} g^< &= \mp ia(p\omega RT)F(p\omega RT) , \\ g^> &= ia(p\omega RT)[1 \mp F(p\omega RT)], \end{aligned} \tag{50}$$

where the spectral function is known from Eq. (49).

Assuming the conventional ansatz, i.e., to replace the ω dependence of the distribution function by its quasiparticle value corresponding to the Wigner distribution function f_W, we get, for $g^<$,

$$g^<(k\omega RT) = \mp ia(k\omega RT)\, f_W(kRT). \tag{51}$$

This is quite good as long as the quasiparticle picture holds true and no memory effects play a role. As we noted from the discussions made following the gauge invariance in Sec. IV, the formulation of kinetic equations with high fields are basically connected with a careful formulation of retardation times. Therefore, the simple ansatz called the KB ansatz, will certainly fail.

Another obscure discrepancy is the fact that with the old ansatz, one has some minor differences in the resulting collision integrals if one compares it with the results from the density operator technique. With the old ansatz, one gets just one-half of all retardation times in the various time arguments [50,5]. This annoying discrepancy had remained obscure until the recent work of Lipavsky *et al.* [61].

Lipavsky *et al.* [61] give an expression for the $g^<$ function in terms of expansion after various times. Using only the first term, we can write, in Wigner coordinates,

$$g^<(p,T,\tau) = \mp if_W\left(p, T - \frac{|\tau|}{2}\right) A(p,\tau,T). \tag{52}$$

The GKB ansatz of Lipavsky is an exact relation if the self-energy is taken in the Hartree-Fock approximation. Taking into account the requirement of gauge invariance (Sec. IV), the GKB ansatz finally reads

$$\begin{aligned} g^<(k,T,\tau) &= \mp if_W\left(k - \frac{eE|\tau|}{2}, T - \frac{|\tau|}{2}\right) A(k,\tau,T) , \\ g^>(k,T,\tau) &= i\left[1 \mp f_W\left(k - \frac{eE|\tau|}{2}, T - \frac{|\tau|}{2}\right)\right] A(k,\tau,T). \end{aligned} \tag{53}$$

In the end, we can use the joint spectral function derived in the Sec. IV B (49) to obtain the resulting ansatz valid for any applied electric field strength

$$\begin{aligned} g^<(k\tau RT) = \mp i\exp&\left[-\frac{i}{\hbar}\left(\epsilon_k\tau + \frac{e^2E^2}{24m}\tau^3\right)\right] \\ &\times f_W\left(k - \frac{eE|\tau|}{2}, R, T - \frac{|\tau|}{2}\right). \end{aligned} \tag{54}$$

In order to get more physical insight into this ansatz we transform it into the frequency representation

$$\begin{aligned} g^<(k\omega RT) = \mp i2\int_0^\infty d\tau \cos\frac{1}{\hbar}&(\hbar\omega\tau - \epsilon(k,R,T)\tau \\ &- \frac{e^2E^2}{24m}\tau^3) f_W\left(k - \frac{eE\tau}{2}, T - \frac{\tau}{2}\right) . \end{aligned} \tag{55}$$

By neglecting the retardation in f_W we recover the ordinary ansatz (51) with the spectral function (49); see also (40). The generalized ansatz takes into account history by an additional memory. This ansatz is superior to the Kadanoff-Baym ansatz in the case of high external fields in several respects [55]: (i) It has the correct spectral properties, (ii) it is gauge invariant, (iii) it preserves causality, (iv) the quantum kinetic equations derived with Eq. (18) coincide with those obtained with the density matrix technique [50], and (v) it reproduces the Debye-Onsager relaxation effect [62] .

V. FIELD-DEPENDENT KINETIC EQUATIONS

With the help of the gauge invariant formulation of Green's function (Sec. IV), we can write the general kinetic equation (14) in the gauge-invariant form

$$i\frac{\partial}{\partial T}f(k,T) + e\mathbf{E}\cdot\nabla_k f(k,T) = \int_0^\infty d\tau\left[\left\{g^>\left(k - \frac{eE}{2}\tau, \tau, T - \frac{\tau}{2}\right), \Sigma^<\left(k - \frac{eE}{2}\tau, -\tau, T - \frac{\tau}{2}\right)\right\}_+ - \{g^<, \Sigma^>\}_+\right], \tag{56}$$

where all functions are now gauge invariant. In Sec. V A we will show the explicit form of the kinetic equation obtained with the help of the derived fluctuation approximation.

A. The generalized random-phase approximation

As already been obtained in Sec. IV, the collective effects caused by long-ranged Coulomb potentials can be described by fluctuating quantities. We obtained the expression of self-energy in the shielded potential approximation for the RPA Eq (35). It is now quite simple to introduce this form in the generalized kinetic equation (56) and consider the gauge-invariant correlation function $g^{\gtrless}$ from (54). The resulting collision integral can be written in the following manner, where we have used a saddle-point approximation for the inner fast oscillating time integration:

$$
\begin{aligned}
I_a(k_a,T) = \sum_b \int_0^\infty d\tau \int \frac{d\mathbf{k}'_a d\mathbf{k}_b d\mathbf{k}'_b}{(2\pi\hbar)^6} \delta\left(\mathbf{k}_a + \mathbf{k}_b - \mathbf{k}'_a - \mathbf{k}'_b\right) \\
\times \{ f(k'^*_a) f(k'^*_b)[1 \mp f(k^*_a)][1 \mp f(k^*_b)] - f(k^*_a) f(k^*_b)[1 \mp f(k'^*_a)][1 \mp f(k'^*_b)] \} \\
\times \mathrm{Re}\Bigg(V^{sr}_{ab}\left[k'_b - k_b, \epsilon\left(k'_b - \frac{e_b E}{2}\tau\right) - \epsilon\left(k_b - \frac{e_b E}{2}\tau\right), T - \tau \right] \\
\times V^{sa}_{ba}\left[k'_a - k_a, \epsilon\left(k'_b - \frac{e_b E}{2}\tau\right) - \epsilon\left(k_b - \frac{e_b E}{2}\tau\right), T \right] \\
\times \exp\left\{ -i[\epsilon(k_a) + \epsilon(k_b) - \epsilon(k'_a) - \epsilon(k'_b)]\tau - \frac{\mathbf{E}}{2}\left(\frac{\mathbf{k}_a e_a}{m_a} + \frac{\mathbf{k}_b e_b}{m_b} - \frac{\mathbf{k}'_a e'_a}{m'_a} - \frac{\mathbf{k}'_b e'_b}{m'_b} \right)\tau^2 \right\} \Bigg).
\end{aligned}
\tag{57}
$$

Again, the shorthand notation $f(k^*) = f(k - eE\tau, T - \tau)$ is used. Furthermore, it was useful to introduce the shielded potential in the following way:

$$
\begin{aligned}
\sum_c (\epsilon^a_{bc})^{-1} V_{ca} &= V^{sA}_{ba} \\
\sum_c V_{ac} (\epsilon^a_{cb})^{-1} &= V^{sR}_{ab}.
\end{aligned}
\tag{58}
$$

We see that the field influences the collision integral by two main points. First, it widens the δ distribution of energy conserving to an oscillating part $\sim \tau^2$ and second, some retardation occurs, which causes a non-Markovian behavior of the collision integral. To summarize, this field-dependent collision integral contains two important effects: (i) collisional broadening, which is a memory effect also existing in the zero field case [52,63], and (ii) the intracollisional field effect, which is determined by field-dependent two-particle dynamics [62,55]. These field effects can be estimated by introducing a relaxation field [64,62,65,66].

The derived Lennard-Balescu collision integral (57) is a generalization of the known one (see, e.g., [12]) to arbitrary external fields as well as including memory effects. The latter one are discussed in another paper [53].

It must be especially pointed out that the *dynamical* behavior in V_s is influenced by the field. If we neglect the center of time retardation in V_s, we get just the Born approximation, but with the static screened potential

$$
\begin{aligned}
I_{ab} = \frac{2}{\hbar^2} \int \frac{d\mathbf{k}'_a d\mathbf{k}_b d\mathbf{k}'_b}{(2\pi\hbar)^9} \delta\left(\mathbf{k}_a + \mathbf{k}_b - \mathbf{k}'_a - \mathbf{k}'_b\right) V_s^2(\mathbf{k}_a - \mathbf{k}'_a) \\
\times \int_0^\infty d\tau 2\cos\frac{1}{\hbar}\left\{ (\epsilon_a + \epsilon_b - \epsilon'_a - \epsilon'_b)\tau - \frac{e\mathbf{E}\tau^2}{2}\left(\frac{\mathbf{k}_a}{m_a} + \frac{\mathbf{k}_b}{m_b} - \frac{\mathbf{k}'_a}{m_a} - \frac{\mathbf{k}'_b}{m_b} \right) \right\} \\
\times \{ f'_a f'_b (1 - f_a)(1 - f_b) - f_a f_b (1 - f'_a)(1 - f'_b) \}_{(\mathbf{k} - e\mathbf{E}\tau, T - \tau)} \cdot
\end{aligned}
\tag{59}
$$

This equation was investigated for high-field problems in [50,67] and was first derived by Levinson [68] for electron phonon interactions by the density operator technique. Other authors have redeveloped this result by the Green's-function technique [69,44]. This field-dependent Boltzmann equation is most commonly used in integral form [70], where it is named the Barker-Ferry equation [5]. In [4] it was shown that this equation reduces to the semiclassical Boltzmann equation for very high field strengths.

Similar kinetic equation were derived for a pulse excited semiconductor transport [7,10,6]. Without the used saddle-point approximation one retains with double time integrations [71]. There an additional damping is introduced to ensure convergence. It can also be found in the generalization of magnetic fields in [72,73]. Further it is interesting to remark that this equation fulfills the global energy conservation, i.e., the sum of the kinetic and the

correlation energy [52,63].

The explicit discussion of the dielectric function using the GKB ansatz is given in [55]. The field dependence of the collision integral (59) leads to a relaxation field diminishing the applied electric field, the quantum result of which is given analytically in [62]. The solution of the field-dependent kinetic equation and its application to high-field transport in plasmas can be found in [74,75]. Among these results an optical bistability can be observed [54]. If the field dependence is switched off, memory effects will remain due to the retardation, which leads to a modification of the transport properties [52,53].

B. The fluctuation-dissipation theorem with fields

Sometimes it is useful to have another form of collision integral. Therefore we do not apply the optical theorem (32) in (35), but rewrite the function $L^{\gtrless}$ in the following way:

$$L^{\gtrless} = \frac{1}{2}(L^{>} + L^{<}) \mp \frac{1}{2}(L^{>} - L^{<}) = \overline{\langle \delta E(1)\delta E(2)\rangle}\frac{\epsilon^2 k^2}{e^2} \mp \mathrm{Im}\, L. \tag{60}$$

Here we made use of the definition of field fluctuations (24). Remembering the relation between the dielectric function and the fluctuation L (33), one recognizes

$$V\ \mathrm{Im}\, L = \mathrm{Im}(\epsilon^R)^{-1}.$$

Finally one finds

$$L^{\gtrless} = [\overline{\langle \delta E(1)\delta E(2)\rangle} \mp \mathrm{Im}(\epsilon^R)^{-1}]\ \frac{1}{V}. \tag{61}$$

It is now possible to express the general collision integral (56) with the help of (35) and (61) in the form

$$I_a(k,T) = \sum_{bc}\int_0^\infty d\tau \int \frac{d\mathbf{p}'d\mathbf{p}''}{(2\pi\hbar)^3}\delta(\mathbf{k}-\mathbf{p}'-\mathbf{p}'')\,V_{aa}(p') \times 2\mathrm{Re}\left\{\exp\left[-i\left([\epsilon_a(k)+\epsilon_a(p'')]\tau - \frac{e_a\mathbf{E}\mathbf{p}'}{2m_a}\tau^2\right)\right]\left[\overline{\langle\delta E\delta E\rangle_{bc}}\left(p',-\tau,T-\frac{\tau}{2}\right)[f_a(p''^*)-f_a(k^*)]\right.\right.$$
$$\left.\left. +\mathrm{Im}(\epsilon^R_{bc})^{-1}\left(p',-\tau,T-\frac{\tau}{2}\right)\{f_a(k^*)[1\mp f_a(p''^*)]+f_a(p''^*)[1\mp f_a(k^*)]\}\right]\right\}. \tag{62}$$

From this form one deduces in thermodynamic equilibrium, in which case the collision integral vanishes, the following condition for fermions or bosons:

$$0 = \int \frac{d\omega}{\sqrt{2\pi b}} g\left(\frac{\omega}{\sqrt{2\pi b}}\right)\left[\overline{\langle\delta E\delta E\rangle}(p',\omega-x)\tanh\left(\frac{x}{2kT}\right) - \mathrm{Im}(\epsilon^R)^{-1}(p',\omega-x)\right] \tag{63}$$

with

$$\sqrt{\frac{2\pi}{b}}\, g\left(\frac{\omega}{\sqrt{2\pi b}}\right) = 2\int_0^\infty \cos(\omega\tau + b\tau^2)d\tau\ , \tag{64}$$

which can be expressed in terms of Fresnel functions [58]. Here b stands for $\frac{e_a\mathbf{E}\cdot\mathbf{p}'}{m_a}$. The field-free limit of (64) is $2\pi\delta(\omega)$ for $b \to 0$. In this case, which coincides with transverse electric fields, (63) leads to the conventional quantum fluctuation-dissipation theorem [76]

$$\overline{\langle\delta E\delta E\rangle}(p',\omega)\tanh\left(\frac{\omega}{2kT}\right) = \mathrm{Im}(\epsilon^R)^{-1}(p',\omega). \tag{65}$$

As is recognizable, the conventional fluctuation-dissipation theorem can be obtained only in the limit of small fields or transversal momentum, where the parameter b vanishes and the function g collapses into the δ distribution. This means that in high fields only integral forms of fluctuation-dissipation theorems can be given.

VI. CONCLUSION

In this paper the fundamental equation of many-particle quantum statistics of nonequilibrium is treated in respect to arbitrary high electric fields. Generalizations are found for the shielded potential approximation valid for any field strength. This results in non-Markovian behavior of the obtained collision integral, also known as intracollisional-field effect, and in a broadening of the energy conservation, the so-called collisional broadening, caused by applied electric fields.

From the shielded potential approximation a generalized Lennard-Balescu equation is derived containing field-dependent screening in the RPA for external electric fields of arbitrary strength. From this equation general fluctuation-dissipation theorems can be concluded only in an integral form, which is also a direct expression for the nonequilibrium situation in high electric fields, even if stationary cases are available.

As a forthcoming work the RPA has to be exceeded for high field transport because it is known from ordinary transport theory that the local field corrections by Hubbard [77] and Singwi *et al.* [78] provide a more realistic description of dielectric properties. The influence of bound states [79,80] should also be included, but this is connected with the treatment of the field-dependent T-matrix calculations. This will be published elsewhere.

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